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Open strings, 2D gravity and AdS/CFT correspondence

Mariano Cadoni[a*], Marco Cavaglià[b†]
[a] *Dipartimento di Fisica, Università di Cagliari,*
Cittadella Universitaria 09042, Monserrato, Italy and INFN, Sezione di Cagliari
[b] *Departamento de Física, Universidade da Beira Interior*
R. Marquês d'Ávila e Bolama, 6200 Covilhã, Portugal.

Abstract

We present a detailed discussion of the duality between dilaton gravity on AdS_2 and open strings. The correspondence between the two theories is established using their symmetries and field theoretical, thermodynamic, and statistical arguments. We use the dual conformal field theory to describe two-dimensional black holes. In particular, all the semiclassical features of the black holes, including the entropy, have a natural interpretation in terms of the dual microscopic conformal dynamics. The previous results are discussed in the general framework of the Anti-de Sitter/Conformal Field Theory dualities.

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*Email: cadoni@ca.infn.it

†Email: cavaglia@mercury.ubi.pt

I. INTRODUCTION

One of the most striking features of the Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence [1] is the possibility of relating physical theories that appear completely different at first sight. Although the meaning of the AdS/CFT correspondence is yet to be fully clarified, we expect it will help shed light on fundamental issues of contemporary theoretical physics, such as the non-perturbative regime of Yang-Mills and string theories.

Lower-dimensional models are often used in theoretical physics as simplified models to investigating complex systems. This approach allows to formulate the problem under investigation in a mathematical simpler context, yet retaining the crucial characteristics of the original model. Applying this strategy to the AdS/CFT correspondence we are lead to investigate the lowest-dimensional, $d = 2$, member of the $\mathrm{AdS}_d/\mathrm{CFT}_{d-1}$ family. In this case the AdS/CFT conjecture states that gravity on AdS_2 is dual to a one-dimensional conformal field theory living on the timelike boundary of AdS_2. Widely investigated in the recent literature, the $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence has, however, revealed itself much more puzzling than its higher dimensional counterparts [2–9]. Specific features of two-dimensional gravity and of the conjectured CFT living on the boundary of AdS_2 conspire indeed to make the whole subject very difficult to analyze. Classical two-dimensional (dilaton) gravity is a conformal theory itself. It can be formulated as a nonlinear sigma-model [10], which, at the classical level, is endowed with conformal symmetry. So we would naively expect gravity on AdS_2 to be dual to a *two-dimensional* CFT. However, it has been shown that the conformal symmetry associated with AdS_2 is infinite dimensional and is generated by a Virasoro algebra. Moreover, there is some evidence that it can be realized in terms of boundary fields describing deformations of the boundary of AdS_2 [3–5]. These results seem to indicate that the conformal theory is actually a one-dimensional CFT, though the search for a viable candidate has not been successful yet. (See Refs. [3,4,7].)

The above features have a strong impact on the study of two-dimensional gravity structures, i.e., black holes, by means of conformal field theory techniques. Previous attempts to calculate the statistical entropy of AdS_2 black holes were only partially successful [3–5] (A mismatch of a factor $\sqrt{2}$ between the thermodynamic and statistical entropy was found). Though the free energy of AdS_2 black holes depends quadratically on the Hawking temperature [11], a feature which is typical of two-dimensional CFTs, it has been shown that AdS_2 black holes are completely characterized by the charges associated with the asymptotic symmetries of AdS_2 [3,4]. These charges are defined on the timelike boundary of AdS_2, suggesting that AdS_2 black holes admit a description in terms of a one-dimensional conformal field theory.

In this paper we make a step forward in clarifying the meaning of the AdS/CFT duality in two dimensions. Starting from the nonlinear sigma model description of two-dimensional dilaton gravity [10] we discuss in depth the duality between two-dimensional dilaton gravity on AdS_2 and open strings. We show that in the weak-coupling regime two-dimensional dilaton gravity on AdS_2 has two different degeneration limits which correspond to Neumann and Dirichlet boundary conditions for the open string, respectively. We put the modes of the gravitational theory on the boundary in a one-to-one correspondence with the string modes and explain the semiclassical properties of the AdS black hole – including the entropy

– in terms of the dual CFT microscopic dynamics. Some results of this paper have been anticipated in a previous letter [12]. Here we extend and complete those results, in particular we present a detailed and systematic discussion of the AdS_2/CFT duality and clarify the meaning of Dirichlet and Neumann boundary conditions. We also speculate on the relevance of our results in the more general framework of higher-dimensional AdS/CFT dualities.

The structure of the paper is as follows. In Section II and Section III we briefly review the main features of two-dimensional dilaton gravity on AdS_2 and its formulation as a nonlinear sigma model, respectively. We also show that in the weak-coupling regime the theory is described by an open bosonic string. In Section IV we compare the symmetries of two-dimensional dilaton gravity on AdS_2 to the symmetries of the string. In Section V we use the previous results and further field theoretical arguments to show that gravity on AdS_2 is dual to the bosonic string. In Section VI we put in a one-to-one correspondence the string modes with the asymptotic modes of AdS_2 gravity. In Section VII we use the AdS/CFT correspondence to explain the semiclassical properties of the AdS_2 black hole in terms of the microscopic conformal dynamics. Finally, in Section VIII we discuss our results.

II. THE TWO-DIMENSIONAL DILATON GRAVITY THEORY

Our starting point is the two-dimensional dilaton gravity action

$$A = \frac{1}{2}\int d^2x\,\sqrt{-g}\,\left(\phi R + V(\phi)\right). \tag{1}$$

The scalar field ϕ is related to the usual definition of the dilaton φ by $\phi = \exp(-2\varphi)$. The two-dimensional model (1) has been widely investigated in the literature [13]. Because of its simplicity it has been used to address fundamental problems of quantum gravity and black hole physics in a mathematically simplified context.

In this paper we restrict attention on dilaton gravity models that have AdS_2 as classical solution. The prototype of these models is the Jackiw-Teitelboim theory [14], JT for short, which is obtained setting $V(\phi) = 2\lambda^2\phi$ in Eq. (1). Although the JT theory may look rather uninteresting – there are no local physical degrees of freedom, the general solution describes a spacetime of constant negative curvature – a closer examination reveals a much richer structure. In particular, the theory admits black hole solutions [11]. In the following we will briefly review the main features of AdS_2 black holes, referring the reader to the vast literature on the subject for a more detailed discussion. (See, e.g., Ref. [11] and references therein.)

Owing to the extended Birkhoff theorem the general solution of the JT model in the Schwarzschild gauge is

$$ds^2 = -\left(\lambda^2 r^2 - \frac{2m_{bh}}{\lambda\phi_0}\right)dt^2 + \left(\lambda^2 r^2 - \frac{2m_{bh}}{\lambda\phi_0}\right)^{-1} dr^2\,, \quad \phi = \phi_0\lambda r\,, \quad m_{bh} \geq 0\,. \tag{2}$$

The general theory (1) admits the existence of the gauge invariant, local conserved quantity [15]

$$M = N(\phi) - g_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi\,, \qquad N(\phi) = \int^\phi d\phi' V(\phi')\,. \tag{3}$$

On the classical orbit M is constant and proportional to the ADM mass of the system [16]. For the JT black hole we have $M = 2\phi_0 \lambda m_{bh}$. For purely dimensional reasons two-dimensional dilaton gravity does not allow a dimensionful analog of the four-dimensional Newton constant. However, ϕ^{-1} represents the (coordinate dependent) coupling constant of the theory. For the JT model, in particular, ϕ_0^{-1} plays the role of a dimensionless Newton constant, G_2. The metric (2) represents different, locally equivalent, parametrization of AdS$_2$ according to the value of m_{bh}. The presence of the scalar field ϕ makes these parametrizations globally inequivalent [11]. In this paper, following the notations of Ref. [11], solutions with $m_{bh} > 0$ and $m_{bh} = 0$ will be denoted by AdS$_2^+$ and AdS$_2^0$, respectively. AdS$_2^+$ can be interpreted as a black hole of mass m_{bh} with a singularity at $r = 0$, a timelike boundary at $r = \infty$, and an event horizon at $r = (2m/\lambda^3 \phi_0)^{1/2}$. AdS$_2^0$ can be considered as the ground state, zero mass solution. In this case the singularity at $r = 0$ is lightlike. Let us stress that the global topology of both AdS$_2^0$ and AdS$_2^+$ is different from the topology of the full AdS$_2$ geometry (the maximally extended spacetime). The latter is a geodesically complete spacetime with cylindrical topology and two timelike boundaries. Because of the singularity at $r = 0$ both AdS$_2^+$ and AdS$_2^0$ are singular spacetimes with a single timelike boundary at $r = \infty$.

Since AdS$_2^+$ and AdS$_2^0$ are locally equivalent, a coordinate transformation exists that maps the solution (2) with $m_{bh} > 0$ into the solution with $m_{bh} = 0$ [11]. Later on this paper we will make use of this coordinate transformation. In the conformal gauge the AdS$_2^0$ metric is

$$ds^2 = \frac{1}{\lambda^2 x^2}(-dt^2 + dx^2)\,. \tag{4}$$

The metric of the AdS$_2^+$ black hole is

$$ds^2 = \frac{a^2}{\sinh^2(a\lambda\sigma)}(-d\tau^2 + d\sigma^2)\,, \tag{5}$$

where $a = (2m_{bh}/\phi_0\lambda)^{1/2}$. The two metrics above are related by the change of coordinates

$$t = \frac{1}{a\lambda} e^{a\lambda\tau} \cosh(a\lambda\sigma)\,, \qquad x = \frac{1}{a\lambda} e^{a\lambda\tau} \sinh(a\lambda\sigma)\,. \tag{6}$$

In the following, we will also use the light-cone coordinates

$$u = \frac{1}{2}(t + x), \qquad v = \frac{1}{2}(-t + x)\,. \tag{7}$$

In this coordinate frame the AdS$_2^0$ solution is

$$ds^2 = \frac{4}{\lambda^2 (u+v)^2} du dv, \qquad \phi = -\phi_0 [\lambda(u+v)]^{-1}. \tag{8}$$

The black hole solution (2) can be interpreted as a thermodynamic system and the usual thermodynamic parameters can be associated to it. The black hole mass depends quadratically on both the Hawking temperature T and the entropy S_{bh} [11],

$$m_{bh} = \frac{2\pi^2\phi_0}{\lambda}\,T^2\,, \qquad S_{bh} = 4\pi\sqrt{\frac{m\phi_0}{2\lambda}}\,. \tag{9}$$

A fundamental question concerns the statistical interpretation of the thermodynamic quantities in Eq. (9). In particular, one would be able to identify the microscopic degrees of freedom whose dynamics produces the huge degeneracy which is contained in Eq. (9).

At the semiclassical level black holes are unstable because of the Hawking effect. In the two-dimensional context the Hawking evaporation process has a simple and nice explanation. From the coordinate transformation (6) we find that the relation between AdS_2^+ and AdS_2^0 is formally equivalent to the relation between Rindler and Minkowski spacetimes. By quantizing a scalar field in the fixed backgrounds defined by AdS_2^+ and AdS_2^0 one finds that the positive frequency modes of the quantum field with respect to Killing vector ∂_t are not positive frequency modes with respect to Killing vector ∂_τ. Hence, the vacuum state which is seen by an observer in the (τ,σ) reference frame appears filled with thermal radiation to an observer in the (t,x) frame. The flux corresponds to a Planck spectrum with temperature given by Eq. (9) [11]. The relation between mass and temperature in Eq. (9) is nothing else but the two-dimensional Stefan-Boltzmann law.

The previous features strongly suggest the existence of an underlying two-dimensional field theory whose microscopic dynamics is responsible for the thermodynamic behavior of the black hole.

III. THE SIGMA MODEL APPROACH TO TWO-DIMENSIONAL DILATON GRAVITY

The dilaton gravity action (1) can be cast in a nonlinear conformal sigma model form [10]. The two-dimensional Ricci scalar $R^{(2)}(g)$ can be locally written as

$$R^{(2)}(g) = 2\,\nabla_\mu A^\mu\,, \qquad A^\mu = \frac{\nabla^\mu\nabla^\nu\chi\nabla_\nu\chi - \nabla_\nu\nabla^\nu\chi\nabla^\mu\chi}{\nabla_\rho\chi\nabla^\rho\chi}\,, \tag{10}$$

where χ is an auxiliary scalar function. Equation (10) can be checked using conformal coordinates and general covariance arguments. Differentiating Eq. (3), setting $\chi=\phi$ in Eq. (10), and integrating per parts, the action (1) can be written as a functional of M and ϕ

$$A = \frac{1}{2}\int_\Sigma d^2x\,\sqrt{-g}\,\frac{\nabla_\mu\phi\nabla^\mu M}{N(\phi)-M}\,. \tag{11}$$

Clearly, the action (11) describes a two-dimensional nonlinear sigma model. In the canonical form, using the metric parametrization

$$g_{\mu\nu} = \rho\begin{pmatrix}\alpha^2-\beta^2 & \beta\\ \beta & -1\end{pmatrix}\,, \tag{12}$$

the super-Hamiltonian and super-momentum are

$$\mathcal{H}_0 = 2[N(\phi)-M]\pi_\phi\pi_M + \frac{1}{2[N(\phi)-M]}\phi' M'\,, \tag{13}$$

$$\mathcal{H}_1 = -\phi'\pi_\phi - M'\pi_M\,. \tag{14}$$

The canonical action must be complemented by a boundary term at the spatial boundaries to make the action finite and differentiable. According to Ref. [17] the boundary term coincides with the conserved charge, i.e., the mass, of the black hole. [See below, Eq. (32)]. The canonical chart $(\phi, \pi_\phi, M, \pi_M)$ is related to the canonical chart $(\phi, \Pi_\phi, \rho, \Pi_\rho)$ by the map

$$\begin{aligned} M &= N(\phi) - \frac{4\rho^2\Pi_\rho^2 - \phi'^2}{\rho}\,, \\ \pi_M &= \frac{\rho^2\Pi_\rho}{4\rho^2\Pi_\rho^2 - \phi'^2}\,, \\ \pi_\phi &= \Pi_\phi - \frac{\rho^2\Pi_\rho}{4\rho^2\Pi_\rho^2 - \phi'^2}\left[V(\phi) + 2\Pi_\rho\left(\frac{\phi'}{\rho\Pi_\rho}\right)'\right]. \end{aligned} \tag{15}$$

Equation (15) proves the equivalence of Eq. (1) and Eq. (11) at canonical level.

Let us consider the JT model. In this case it is convenient to define the "coupling constant" field

$$\psi = -\frac{1}{2\lambda^2\phi}\,. \tag{16}$$

In terms of M and ψ the JT action is

$$A = \int d^2x\,\sqrt{-g}\,\partial_\mu M \partial^\mu \psi \cdot \frac{1}{1 - 4\lambda^2\psi^2\, M}\,. \tag{17}$$

The boundary of the spacetime is now located at $\psi = 0$. The action (17) can be expanded around $\psi = 0$,

$$A = \int d^2x\,\sqrt{-g}\,\partial_\mu M \partial^\mu \psi \left[1 + \sum_{k=1}^{+\infty}(2\lambda)^{2k} M^k \psi^{2k}\right]. \tag{18}$$

Equation (18) is both a weak-coupling expansion in terms of the coordinate-dependent gravitational coupling of the model and an expansion near the boundary of AdS_2. This fact suggests that the gravitational theory can be represented perturbatively as an expansion around the boundary. The first term (zero order) of the expansion coincides with the action for a bosonic string living in a two-dimensional flat target spacetime and describes the (off-shell) weak-coupled gravitational theory. It can be cast in the usual form (see Ref. [18] for notations)

$$\begin{aligned} A_0 &= \frac{1}{2\pi\alpha'}\int d^2z \partial X^\mu \bar{\partial} X_\mu \\ &= \frac{1}{2\pi\alpha'}\int d^2z(-\partial X^0 \bar{\partial} X^0 + \partial X^1 \bar{\partial} X^1) \\ &= \frac{1}{2\pi\alpha'}\int d^2z(\partial X^2 \bar{\partial} X^2 + \partial X^1 \bar{\partial} X^1)\,, \end{aligned} \tag{19}$$

by defining the new fields

$$\begin{aligned} \sqrt{\pi\alpha'}M &= \frac{1}{2}(X^1 + iX^2) = \frac{1}{2}(X^1 + X^0)\,, \\ \sqrt{\pi\alpha'}\psi &= \frac{1}{2}(X^1 - iX^2) = \frac{1}{2}(X^1 - X^0)\,, \end{aligned} \tag{20}$$

where $\sqrt{\alpha'}$ is the string length, and

$$z \equiv u = \frac{1}{2}(\sigma^1 + i\sigma^2)\,, \qquad \bar{z} \equiv v = \frac{1}{2}(\sigma^1 - i\sigma^2)\,. \tag{21}$$

Higher orders in the expansion (18) can be interpreted as interaction terms for the bosonic string (19). They describe perturbative (off-shell) effects induced by the gravitational bulk on the boundary. Classically, the two-dimensional JT model is a topological theory with no propagating physical degrees of freedom. Owing to the Birkhoff theorem [10] the physics on the gauge shell, i.e., in the fundamental state, is completely determined by the spacetime boundary where the conserved charge is defined, whereas the bulk is pure gauge. In the perturbative sigma model approach the first term of the expansion (18) can be interpreted as describing both the (off-shell) gravitational theory on the boundary and the weak-coupling regime of the theory. Therefore, we expect that the free bosonic string (19) – with properly fixed boundary conditions – describes the semiclassical properties of the theory. Higher orders in the coupling constant perturbative expansion (18) describe the corrections to the off-shell dynamics on the boundary and lead, in the quantum theory, to gravitational corrections to the classical geometry. (See e.g. Ref. [19] where quantum corrections to the ADM mass of the Schwarzschild black hole have been calculated at the second order in the curvature expansion.)

The boundary conditions to be imposed on the free bosonic string (19) are essential in determining the physical content of the AdS/CFT correspondence. First of all, we note that AdS_2 has a timelike boundary at $x = 0$, so the CFT (19) must necessarily describe open strings. Since open strings propagating in a two-dimensional target spacetime do not have transverse excitations, we can impose either Dirichlet boundary conditions $[\partial_a X^\mu(x=0) = 0]$ or Neumann boundary conditions $[n^a \partial_a X^\mu(x=0) = 0$, where n^a is the normal to the boundary]. Expanding the fields on the boundary [see below Eq. (33)], in the former case we have

$$X^\mu(x=0) = \sqrt{\pi\alpha'} M_0(t) = \text{constant}\,, \tag{22}$$

where $M_0(t)$ is the (constant) zero-mode of the mass field M on the boundary. Dirichlet boundary conditions break translation invariance. Moreover, they hold fixed the endpoint of the string on the boundary and do not allow any dynamical degree of freedom on the boundary itself. Hence, Dirichlet boundary conditions realize a $\mathrm{AdS}_2/\mathrm{CFT}_2$ correspondence. The one-dimensional boundary can be interpreted as a D-brane (0-brane). Possibly, a non-trivial dynamics on the brane can be generated by the introduction of Chan-Paton factors. (See, e.g., Ref. [18].)

Neumann boundary conditions do not break translation invariance. They allow for excitations on the boundary,

$$X^\mu(x=0) = \sqrt{\pi\alpha'} M_0(t)\,. \tag{23}$$

Since Neumann boundary conditions allow dynamical degrees of freedom on the boundary, they seem to realize a $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence, where CFT_1 is a genuine one-dimensional CFT generated by the charges living on the boundary [3,4].

In addition to the timelike boundary at $x = 0$, AdS_2^0 has an inner null boundary. However, the presence of the latter does not influence the dynamics of the open string. In the conformal coordinate frame (t, x) the metric of AdS_2^0 is given by Eq. (4) and the presence of the dilaton requires

$$-\infty < t < \infty\,, \qquad 0 \leq x < \infty\,. \tag{24}$$

In this coordinate frame the inner null boundary is located at $x = \infty$. Eq. (4) implies that AdS_2^0 is conformal to the Minkowski spacetime. Hence, because of conformal invariance open strings on AdS_2^0 are equivalent to open strings on the region of the (t, x) Minkowski spacetime defined by Eq. (24). In the next section we will discuss how the symmetries of the bosonic string (19) reflect in the asymptotic symmetries of the two-dimensional gravitational theory (1).

IV. SYMMETRIES OF TWO-DIMENSIONAL GRAVITY AND SYMMETRIES OF THE STRING

AdS_2 is a maximally symmetric space, so the JT theory admits three Killing vectors that generate the $SO(1,2) \sim SL(2,R)$ group of isometries. In the JT theory the presence of the dilaton actually breaks the $SL(2,R)$ symmetry [5]. However, this is irrelevant for the present discussion. Indeed, in the weak-coupling regime, $\psi \to 0$, one naturally expects the $SL(2,R)$ symmetry to be enlarged to the full asymptotic symmetry group of AdS_2. Since Eq. (18) is a near-boundary expansion, the only relevant symmetries are the symmetries that leave the AdS_2 metric asymptotically invariant.

The symmetries of the bosonic string (19) are related to the asymptotic symmetry group of AdS_2. The latter has been studied in detail in Refs. [3,4]. The asymptotic symmetries of AdS_2 can be found by imposing suitable boundary conditions for the metric at $r \to \infty$. These boundary conditions express the intuitive notion of "asymptotically anti-de Sitter" and allow the charges associated with the symmetry to be properly defined.

In the Schwarzschild gauge the boundary conditions to be imposed on the metric $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ and on the scalar field ϕ are [3,4]

$$\begin{aligned} g_{tt} &= -\lambda^2 r^2 + \gamma_{tt} + O\left(\frac{1}{r}\right)\,, \\ g_{tr} &= \frac{\gamma_{tr}}{\lambda^3 r^3} + O\left(\frac{1}{x^5}\right)\,, \\ g_{rr} &= \frac{1}{\lambda^2 r^2} + \frac{\gamma_{tt}}{\lambda^4 r^4} + O\left(\frac{1}{r^6}\right)\,, \end{aligned} \tag{25}$$

and

$$\phi = \phi_0 \left[\lambda r + \rho \lambda r + \frac{\gamma_{\phi\phi}}{\lambda r} + O\left(\frac{1}{r^2}\right)\right]\,, \tag{26}$$

respectively. In the previous equations the γ's and ρ are arbitrary functions of t and can be thought as characterizing the deformations of the boundary of AdS_2 and of the dilaton field. In the conformal gauge the boundary of AdS_2 is located at $u = -v$ and the above conditions (at the order k) read

$$
\begin{aligned}
g_{uu} &= U_0(u-v) + \ldots + U_k(u-v)(u+v)^k + O[(u+v)^{k+1}] \\
g_{uv} &= \frac{2}{\lambda^2(u+v)^2} + Y_0(u-v) + \ldots + Y_k(u-v)(u+v)^k + O[(u+v)^{k+1}] \\
g_{vv} &= V_0(u-v) + \ldots + V_k(u-v)(u+v)^k + O[(u+v)^{k+1}] \\
\phi &= -\phi_0\left[\frac{\omega_{-1}}{\lambda(u+v)} + \omega_1\lambda(u+v) + \ldots + \omega_k\lambda^k(u+v)^k + O[(u+v)^{k+1}]\right]
\end{aligned}
\tag{27}
$$

where the coefficients $\Theta_k = (U_k, V_k, Y_k, \omega_k)$ are arbitrary functions of $u-v$. By definition the leading terms in Eqs. (27) are invariant under the transformations generated by the asymptotic symmetry group. The functions Θ_k change according to a representation of the asymptotic symmetry group. Solving the Killing equations for the metric (27), we find that the asymptotic symmetry group is generated by the Killing vectors

$$
\chi^{AdS} = \chi^u(u,v)\partial_u + \chi^v(u,v)\partial_v\,, \tag{28}
$$

where

$$
\chi^u = \frac{1}{2}\left[\epsilon + \epsilon'(u+v) + \frac{1}{2}\epsilon''(u+v)^2\right] + \alpha^u\,, \quad \chi^v = \frac{1}{2}\left[-\epsilon + \epsilon'(u+v) - \frac{1}{2}\epsilon''(u+v)^2\right] + \alpha^v\,. \tag{29}
$$

Here, ϵ is an arbitrary function of $u-v$, $'$ denotes differentiation with respect to $u-v$, and $\alpha^{u,v} = \sum_{k=3}^{+\infty}\alpha_k^{u,v}(u-v)(u+v)^k$. The functions $\alpha^{u,v}$ represent "pure gauge" diffeomorphisms of the two-dimensional gravitational theory that fall off rapidly on the boundary. Expanding the function $\epsilon(u-v)$ in power series, the Killing vectors (28) are recognized to define a conformal group which is generated by the Virasoro algebra

$$
[L_m^{AdS}, L_n^{AdS}] = (m-n)L_{m+n}^{AdS}\,. \tag{30}
$$

The boundary fields Θ_k span a representation of the conformal group. Their transformation law is

$$
\delta_\epsilon\Theta_k = \epsilon\Theta_k' + (h+k)\epsilon'\Theta_k + \ldots\,, \tag{31}
$$

where dots denote terms that depend on higher derivatives of ϵ and on pure gauge diffeomorphisms, and $h=2$ for U_k, V_k, Y_k and $h=0$ for ω_k, respectively. Note that the pure gauge transformations affect the boundary fields but leave invariant the charge associated with the falloff conditions,

$$
J(\epsilon) = \epsilon\lambda\phi_0\left[2\omega_1 + \frac{1}{8}(U_0 + V_0 + 2Y_0)\right] = \epsilon\frac{M_0(t)}{2\lambda\phi_0}\,, \tag{32}
$$

where $J(\epsilon)$ has been calculated for $\omega_{-1} = 1$. Both the mass functional M and the coupling constant field ψ can be expanded in power series around the boundary

$$
M = \sum_{k=0}^{+\infty} M_k(u-v)(u+v)^k\,, \qquad \psi = \sum_{k=1}^{+\infty}\psi_k(u-v)(u+v)^k\,. \tag{33}
$$

Using Eqs. (3) and (16) both M_k and ψ_k can be expressed in terms of the boundary fields. They transform according to Eq. (31) with $h = 0$. The two-dimensional dilaton gravity action or, alternatively, the sigma model action can be expanded around the boundary as well. Expanding in power series the Lagrangian density, $\mathcal{L} = \sum_{k=0}^{+\infty} \mathcal{L}_k(u-v)(u+v)^k$, we find that $\mathcal{L}_k$ transform according to Eq. (31) with $h = 2$, as is expected for a two-dimensional conformal field theory.

The sigma model action (11) is classically invariant under the conformal transformations of the two-dimensional world-sheet. This invariance is not manifest in its two-dimensional gravitational counterpart (1). Conformal invariance of two-dimensional dilaton gravity is more subtle and can be understood in terms of the Weyl-rescaling invariance of the target space coordinates ψ and M [20]. At the leading order in the $\psi \to 0$ expansion the conformal symmetry of the sigma model is the usual two-dimensional conformal symmetry group of the free bosonic string (19) which is generated by the Killing vectors

$$\chi^{CFT} = \chi(z)\,\partial + \tilde{\chi}(\bar{z})\,\bar{\partial}\,. \tag{34}$$

The transformation law of a generic CFT_2 field $X(z,\bar{z})$ of weights $(h,\tilde{h})$ is

$$\delta_{\chi,\tilde{\chi}} X = (\chi\partial + h\partial\chi)X + (\tilde{\chi}\bar{\partial} + \tilde{h}\bar{\partial}\tilde{\chi})X. \tag{35}$$

Expanding $\chi(z)$ and $\tilde{\chi}(\bar{z})$ as

$$\chi(z) = \sum_{m=-\infty}^{+\infty} \gamma_m z^{-m+1}\,, \qquad \tilde{\chi}(\bar{z}) = \sum_{m=-\infty}^{+\infty} \tilde{\gamma}_m \bar{z}^{-m+1}\,, \tag{36}$$

we have

$$\chi^{CFT} = \sum_{m=-\infty}^{+\infty} \left(\gamma_m L_m^{CFT} + \tilde{\gamma}_m \tilde{L}_m^{CFT}\right)\,, \tag{37}$$

where

$$L_m^{CFT} = z^{-m+1}\partial\,, \qquad \tilde{L}_m^{CFT} = \bar{z}^{-m+1}\bar{\partial}\,, \tag{38}$$

each satisfy the Virasoro algebra (30). Finally, the stress-energy tensor is

$$T_{zz} = -\frac{1}{2\pi\alpha'}\partial X^\mu \partial X_\mu = -2\partial M \partial \Psi = \frac{1}{2\pi}\sum_{m=-\infty}^{+\infty} L_m^{CFT} z^{-2-m}\,. \tag{39}$$

In the next section we will see that the asymptotic symmetry group of AdS_2 (with fixed pure gauge diffeomorphisms) coincides with the conformal symmetry group of the free bosonic string with properly chosen boundary conditions.

V. DUALITY OF TWO-DIMENSIONAL GRAVITY ON ADS$_2$ AND OPEN STRINGS

The duality between gravity on AdS_2 and the open string can be realized by putting in a one-to-one correspondence the symmetries of the string and the asymptotic symmetries of AdS_2. The physical content of the AdS_2/CFT correspondence varies according to the boundary conditions that are chosen for the bosonic string (19). We will consider first Dirichlet and then Neumann boundary conditions.

A. Dirichlet boundary conditions

Equations (29) suggest that the u and v components of the Killing vectors of the asymptotic symmetry group, χ^u and χ^v, are not independent. Let us neglect initially the pure gauge diffeomorphisms in Eqs. (29) and define two auxiliary functions $\chi(u)$ and $\tilde{\chi}(v)$ that satisfy the relation (notations will be clear soon)

$$\chi((t+x)/2)|_{x=0} = -\tilde{\chi}((-t+x)/2)|_{x=0} = \frac{1}{2}\epsilon(t)\,. \tag{40}$$

At the second order in the expansion the u and v components of the Killing vectors (28) can be recognized to be the first three terms of the expansion around $x=0$ of the functions $\chi(u)$ and $\tilde{\chi}(v)$

$$\chi = \frac{1}{2}\sum_{k=0}^{+\infty}\frac{1}{k!}\frac{d^k\epsilon}{d(u-v)^k}(u+v)^k\,, \qquad \tilde{\chi} = -\frac{1}{2}\sum_{k=0}^{+\infty}(-1)^k\frac{1}{k!}\frac{d^k\epsilon}{d(u-v)^k}(u+v)^k\,. \tag{41}$$

Assuming that the equivalence is valid at any order and taking into account Eq. (21), we find that the CFT_2 Killing vectors (34) coincide with the AdS_2 Killing vectors (28) where the gauge diffeomorphisms have been fixed as

$$\alpha^u_k = (-1)^{k+1}\alpha^v_k = \frac{1}{2k!}\frac{d^k\epsilon}{d(u-v)^k}. \tag{42}$$

Hence, the asymptotic symmetry group of AdS_2 coincides with the symmetry group of the bosonic open string (19). In order to obtain the second one from the first one we need to fix the gauge diffeomorphisms of the gravitational theory. This gives a non trivial relation between the diffeomorphisms of the two-dimensional dilaton gravity theory and the diffeomorphisms of the conformal field theory.

The previous equations follow from the Dirichlet condition for the function $\mathcal{X}(u,v) \equiv \chi(u)+\tilde{\chi}(v)$, i.e.,

$$\mathcal{X}(u,v) \equiv \chi(u)+\tilde{\chi}(v) = 0\,, \tag{43}$$

on the boundary $u+v=0$. This equation implies that the conformal symmetry is generated by a single copy of the Virasoro algebra and the weights $(h,\tilde{h})$ appear in the transformation law (35) only in the combination $w=h+\tilde{h}$.

The correspondence between the conformal group of the bosonic string and the asymptotic AdS_2 group can be observed directly on the AdS_2 fields Θ_k, M_k, ψ_k and $\mathcal{L}_k$. Each of these fields can be interpreted as the coefficient of the expansion of the corresponding CFT_2 field around the boundary with given weight $w=h+\tilde{h}$ and pole of order p. We have the table:

AdS$_2$ Field	h	$\tilde{h}$	w	p
U_k	2	0	2	0
V_k	0	2	2	0
Y_k	1	1	2	-2
ω_k	0	0	0	-1
M_k	0	0	0	0
ψ_k	0	0	0	1
$\mathcal{L}_k$	1	1	2	0

(44)

The table above represents the AdS$_2$/CFT$_2$ correspondence in terms of fields: the AdS$_2$ fields are interpreted as objects of the two-dimensional CFT. Using Eq. (35) together with Eqs. (41) we recover the transformation (31) of the AdS$_2$ fields.

The AdS$_2$/CFT$_2$ correspondence allows to determine the Virasoro generators of the asymptotic symmetry group of AdS$_2$ from CFT$_2$. Let us consider the Virasoro generators (38). Using the (t, x) coordinates we have

$$L_m^{CFT} = 2^{m-1} \sum_{k=0}^{+\infty} t^{-m+1-k} x^k \beta_{m-2,k} (\partial_t + \partial_x) \,, \tag{45}$$

where

$$\beta_{m-2,k} = \binom{1-m}{k} \,. \tag{46}$$

A similar expression holds for $\tilde{L}_m^{CFT}$. We have seen that the asymptotic symmetry group of AdS$_2$ can be obtained from the conformal group of the string by imposing Eq. (43). Applying the condition (43) to Eq. (37) we find $\gamma_m = (-1)^m \tilde{\gamma}_m$. From Eq. (45) the Killing vectors for the Dirichlet boundary conditions read

$$\chi^{CFT} = \sum_{m=-\infty}^{+\infty} \gamma_m L'_m \,, \tag{47}$$

where

$$L'_m = 2^m \sum_{k=0}^{+\infty} \left[\beta_{m-2,2k} t^{-m+1-2k} x^{2k} \partial_t + \beta_{m-2,2k+1} t^{-m-2k} x^{2k+1} \partial_x \right] \,. \tag{48}$$

The Killing vectors (47) coincide with the Killing vectors of the asymptotic symmetry group of AdS$_2$ (28) with fixed pure gauge diffeomorphisms. [See Eq. (42).] Indeed, let us consider Eq. (29) with fixed gauge diffeomorphisms and expand $\epsilon(t)$ in power series

$$\epsilon(t) = \sum_{m=-\infty}^{+\infty} 2^m \epsilon_m t^{-m+1} \,. \tag{49}$$

The Killing vectors (28) are cast in the form

$$\chi^{AdS} = \sum_{m=-\infty}^{+\infty} \epsilon_m L_m^{AdS} \,, \tag{50}$$

where

$$L_m^{AdS} = 2^m \left\{ \left[t^{-m+1} + \frac{1}{2}(-m+1)(-m)t^{-m-1}x^2 + \ldots \right] \partial_t + \left[(-m+1)t^{-m}x + \ldots \right] \partial_x \right\}$$
$$= L'_m \,. \tag{51}$$

Setting $\epsilon_m = \gamma_m$ the Killing vectors of the AdS_2 asymptotic symmetry group and the Killing vectors of the string with Dirichlet boundary conditions coincide. Therefore, they generate both the full symmetry group of the bosonic open string and the gravitational asymptotic symmetry group of AdS_2 with fixed pure gauge diffeomorphisms. The Virasoro generators of the AdS_2 asymptotic symmetry group are

$$L_m^{AdS} = 2^{m-1} \left\{ \left[(t+x)^{-m+1} + (t-x)^{-m+1} \right] \partial_t + \left[(t+x)^{-m+1} - (t-x)^{-m+1} \right] \partial_x \right\} . \tag{52}$$

The Virasoro generators (52) are simply obtained from the Virasoro generators of the string by taking the linear combination $L_m^{AdS} = L_m^{CFT} + (-1)^m \tilde{L}_m^{CFT}$ and changing coordinates to (t, x). This relation implies that the symmetries of the open string are generated by a single copy of the Virasoro algebra. On the boundary the Virasoro generators are $L_m^{AdS}|_{x=0} = 2^m t^{-m+1}\partial_t$.

It should be noted that both the Killing vectors and the Virasoro generators are now defined outside the spacetime boundary. By fixing the gauge diffeomorphisms we select a subgroup of the full two-dimensional diffeomorphism group of the gravitational theory. This subgroup is recognized to be the conformal group of the string with Dirichlet boundary conditions. The latter can also be defined as a subgroup of the diffeomorphisms that leave the two-dimensional metric asymptotically invariant (AdS_2/CFT_2 duality).

Finally, let us conclude this section with a few equations that will be useful in the following. The subalgebra $SL(2, R)$ of the Virasoro algebra is generated by

$$L_0^{AdS} = t\partial_t + x\partial_x \,, \qquad L_1^{AdS} = 2\partial_t \,, \qquad L_{-1}^{AdS} = \frac{1}{2}(t^2 + x^2)\partial_t + xt\partial_x \,. \tag{53}$$

The Virasoro generator L_0^{AdS} does not generate translations in t but dilatations. Changing coordinates to (τ, σ) [see Eq. (6)] the Virasoro generators become

$$L_m^{AdS} = (2a\lambda)^m e^{-ma\lambda\tau} \frac{1}{a\lambda} \left[\cosh(ma\lambda\sigma)\partial_\tau - \sinh(ma\lambda\sigma)\partial_\sigma \right] . \tag{54}$$

In this reference frame L_0^{AdS} generates translations in the new time coordinate τ.

B. Neumann boundary conditions

Let us now discuss the AdS_2/CFT duality when Neumann boundary conditions are enforced. Imposing the Neumann boundary conditions on the function $\mathcal{X}(u, v)$ [see Eq. (43)] we have, on the boundary $u + v = 0$,

$$\partial_u \chi(u) + \partial_v \tilde{\chi}(v) = 0 \,. \tag{55}$$

Equation (55) is solved by the condition

$$\chi((t+x)/2)|_{x=0} = \tilde{\chi}((-t+x)/2)|_{x=0} = \frac{1}{2}\epsilon(t)\,. \tag{56}$$

Using Eq. (56) we try and put in a one-to-one correspondence the symmetry group of the open string with Neumann boundary conditions with the asymptotic symmetry group of AdS_2. Imposing the condition (56) on the CFT Killing vectors (37) we find $\gamma_m = -(-1)^m \tilde{\gamma}_m$. The Virasoro generators of the asymptotic AdS_2 group with Neumann boundary conditions are

$$L_m^{AdS} = 2^{m-1}\left\{\left[(t+x)^{-m+1} - (t-x)^{-m+1}\right]\partial_t + \left[(t+x)^{-m+1} + (t-x)^{-m+1}\right]\partial_x\right\}\,. \tag{57}$$

From the previous equation it follows that the Virasoro generators vanish on the boundary. Hence, the asymptotic symmetry group of AdS_2 cannot be put in correspondence with the conformal symmetry group of the open string when Neumann boundary conditions are enforced. We will see in the next section that this is due to the impossibility of realizing the symmetry in terms of local string oscillators. Neumann boundary conditions lead to a topological theory without local degrees of freedom and the AdS_2 asymptotic symmetry group can be realized uniquely by the charges [3,4]. In this case the AdS_2/CFT correspondence is local/topological.

VI. MODE EXPANSION AND HOLOGRAPHY

The AdS_2/CFT correspondence can be realized using local oscillator degrees of freedom as well. Let us expand the string field in normal modes

$$X^\mu = x^\mu - ip^\mu \log|z|^2 + i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{m=-\infty}^{+\infty}\frac{1}{m}\left(\alpha_m^\mu z^{-m} + \tilde{\alpha}_m^\mu \bar{z}^{-m}\right)\,. \tag{58}$$

Substituting the previous expansion in Eqs. (20) and comparing the result to the expansion of the fields M and ψ

$$M = \sum_{k=0}^{+\infty}\sum_{m=-\infty}^{+\infty} M_{k,m}x^k t^m\,, \qquad \psi = \sum_{k=1}^{+\infty}\sum_{m=-\infty}^{+\infty}\psi_{k,m}x^k t^m\,, \tag{59}$$

we find (we assume $t>0$ for simplicity)

$$\begin{aligned}\alpha_m^\mu &= -i\sqrt{\pi}2^{-1/2-m}\left[mM_{0,-m} - M_{1,-1-m} \pm \psi_{1,-1-m}\right]\,,\\ \tilde{\alpha}_m^\mu &= -i\sqrt{\pi}2^{-1/2-m}(-1)^m\left[mM_{0,-m} + M_{1,-1-m} \mp \psi_{1,-1-m}\right]\,,\end{aligned} \tag{60}$$

where the $\pm$ signs refer to the 0 and 1 components of α_m^μ, respectively. Equation (60) puts the modes of the string in a one-to-one correspondence with the "gravitational" modes of the fields M and ψ.

Let us now enforce Dirichlet and Neumann boundary conditions on the string field. Neumann boundary conditions imply $M_{k,m}=0$, $\psi_{k,m}=0$ for $k\geq 1$, so Eq. (60) becomes

$$\alpha_m^\mu = (-1)^m\tilde{\alpha}_m^\mu = -i\sqrt{\pi}2^{-1/2-m}\left(mM_{0,-m}\right)\,. \tag{61}$$

The generators of the Virasoro algebra of CFT_2 vanish identically,

$$L_m^{CFT} = \frac{1}{2}\sum_{n=-\infty}^{+\infty} \alpha^\mu_{m-n}\alpha_{\mu n} = 0\,. \tag{62}$$

Therefore, the AdS_2/CFT duality cannot be realized in terms of local oscillators. This result has a natural interpretation. The gravitational theory with Neumann boundary conditions is a topological theory with no local degrees of freedom. The two-dimensional CFT action depends only on $M_{1,-m}$ and $M_{k,m}$, $\psi_{k,m}$ with $k > 1$, so vanishes at any order of the expansion.

The Dirichlet boundary conditions imply

$$p^\mu = 0\,, \qquad \alpha^\mu_m = (-1)^{m+1}\tilde\alpha^\mu_m \qquad \to \qquad M_{0,m} = 0 \quad \text{for} \quad m \neq 0\,, \tag{63}$$

and Eq. (60) becomes

$$\alpha^\mu_m = i\sqrt{\pi}2^{-1/2-m}\left[M_{1,-1-m} \mp \psi_{1,-1-m}\right]\,. \tag{64}$$

The Virasoro generators of CFT_2 are

$$L_m^{CFT} = \frac{1}{2}\sum_{n=-\infty}^{+\infty} \alpha^\mu_{m-n}\alpha_{\mu n} = -\pi 2^{-m}\sum_{n=-\infty}^{+\infty} M_{1,-1-n}\psi_{1,-1-m+n}\,. \tag{65}$$

The previous results have two important consequences. Firstly, we see that the mass field M is constant on the boundary. This is due to the breaking of translational invariance on the boundary that follows from the Dirichlet conditions. [See also Eq. (22).] $M_{0,0}$ is essentially the conserved charge and does not appear in the definition of the string modes. Secondly, with a bit of algebra it can be proved that the gravitational modes $M_{k,m}$ and $\psi_{k,m}$ satisfy the two recurrency relations

$$M_{k+2,m-2} = \frac{m(m-1)}{(k+2)(k+1)}M_{k,m}\,, \qquad \psi_{k+2,m-2} = \frac{m(m-1)}{(k+2)(k+1)}\psi_{k,m}\,. \tag{66}$$

Substituting Eq. (63) in the recurrency relations above, and recalling that for the AdS_2 geometry $\psi_{0,m} = 0$, the gravitational modes are

$$\begin{array}{lllll} k & = & \text{even} & : & M_{k,m} = 0\,, & \psi_{k,m} = 0\,; \\ k & = & \text{odd} & : & M_{k,m} \equiv M_{k,m}(M_{1,m},\psi_{1,m})\,, & \psi_{k,m} \equiv \psi_{k,m}(M_{1,m},\psi_{1,m})\,. \end{array} \tag{67}$$

The gravitational modes $M_{k,m}$ and $\psi_{k,m}$ are completely determined by $M_{1,m}$ and $\psi_{1,m}$ which are, in turn, determined by the string modes through Eq. (64). Therefore, the modes of the bosonic string determine completely the sigma model, i.e., the full gravitational theory. The latter can be expressed as a function of the gravitational modes $M_{k,m}$ and $\psi_{k,m}$ by the perturbative expansion (18). Then, owing to Eq. (64) and Eq. (67), the perturbative expansion (18) can be written as a function of the string modes. We conclude that gravity on AdS_2 is completely determined by the (interacting) Dirichlet open string. Vice versa, the asymptotic two-dimensional gravitational modes near the boundary, that describe boundary deformations, determine completely CFT_2. This is a sort of holographic principle: the physics on the spacetime boundary determines the properties of the theory in the bulk. However, it should be stressed that we have here a somehow unusual realization of the holographic principle. Usually, in the context of the AdS/CFT correspondence we have a gravitational theory defined on a d-dimensional bulk which is dual to a CFT theory living on its $(d-1)$-dimensional boundary. In our case the picture is reversed: The CFT open string lives on the two-dimensional bulk, whereas the gravitational theory is completely defined by a boundary theory. We have already pointed out that this property is related to the peculiar nature of gravity in two spacetime dimensions, which is itself a conformal field theory.

The free bosonic string describes the off-shell dynamics of the classical black hole with mass m_{bh} (the fundamental state of the theory). Higher order terms in the expansion (18) – the interaction vertices of the string – describe the off-shell corrections to the fundamental state due to effects of the bulk. This result is a natural consequence of the topological nature of the theory. It holds in the quantum theory as well, where the (on-shell) fundamental state of a black hole with given classical mass m_{bh} is given by the eigenstate of the mass operator, whereas the sigma model describes the (off-shell) black hole excitations (Quantum Birkhoff Theorem [10]). We recover the result that have previously anticipated: The Dirichlet bosonic string describes the off-shell semiclassical properties of the theory whereas the interaction terms in the perturbative expansion describe higher order corrections to the off-shell dynamics that lead, in the quantum formalism, to the gravitational corrections of the classical geometry.

It is worth noticing that the degrees of freedom involved in the correspondence (64) are local, pure gauge, degrees of freedom. Both two-dimensional dilaton gravity and string theory with two-dimensional target spacetime are topological theories. So the gravitational modes $M_{1,k}, \psi_{1,k}$ and the string modes α^ν_k describe pure gauge degrees of freedom. Since we are dealing with pure gauge degrees of freedom, criticisms could be raised about the relevance of the correspondence that we are discussing. In the next section we will use the correspondence to calculate the statistical entropy of two-dimensional black holes. The reader might argue that we are not counting physical states of the two-dimensional bulk theory. However, this approach is consistent as long as we restrict our discussion to topological theories such as two-dimensional dilaton gravity and three-dimensional pure gravity theories [21,22]. Our treatment of two-dimensional dilaton gravity and that of Refs. [21,22] suggest the existence of a nontrivial relation between local pure gauge degrees of freedom on the bulk and topological degrees of freedom on the boundary. In the three-dimensional case Carlip has found an explicit realization of this relation [21]. In our case we have not been able to find a similar relation, yet the duality that we have found provides a strong evidence in this direction.

The discussion of this section has profound implications on the statistical interpretation of the thermodynamic quantities in Eq. (9). It is commonly believed that the thermodynamic relations (9) hold within some sort of semiclassical approximation to gravity. If the free open string really describes semiclassical AdS_2 gravity then it should provide a statistical description of the thermodynamic relations (9). This is indeed the case, as we will see in the next section.

VII. TWO-DIMENSIONAL BLACK HOLES AS OPEN STRINGS

The AdS_2/open string duality discussed in the previous sections allows to interpreting the excitations of the gravitational theory, i.e., of the black holes, in terms of the excitations of the string. Naively, we would be tempted to use the equations of Sections V and VI to work out the correspondence explicitly. For instance, we could try and use Eq. (65) to calculate the mass of the string state which is associated with the gravitational excitations described by the modes $M_{1,k}, \psi_{1,k}$. Unfortunately, we do not know how to relate explicitly the sigma model modes with the physical parameters of the black hole. So most of the equations of the previous sections cannot be employed to describe black holes straightforwardly. However, the knowledge of the exact form of the correspondence is not necessary. The bare fact that a two-dimensional black hole has a dual description in terms of a two-dimensional conformal field theory with central charge c is sufficient to explain the semiclassical behavior of the black hole.

The energy of the CFT excitation is given by the eigenvalue, m_{CFT}, of the Virasoro operator L_0^{CFT},

$$m_{bh} = \lambda m_{CFT}\,. \tag{68}$$

(With our conventions m_{CFT} is dimensionless.) The energy-temperature and entropy-mass relations of a two-dimensional CFT are [24],

$$m_{CFT} = \frac{\pi}{12}\alpha' c T^2\,, \qquad S_{CFT} = 2\pi\sqrt{\frac{c\, m_{CFT}}{6}}\,, \tag{69}$$

respectively. The previous equations reproduce the functional behavior of the thermodynamic parameters of the black hole, Eq. (9). In order to show that the thermodynamic behavior of the two-dimensional black hole has a direct interpretation in terms of the microscopic dynamics of the two-dimensional CFT, we must show that Eqs. (69) match Eqs. (11) exactly. This can be done by expressing the central charge c associated with the central extension of the Virasoro algebra generated by L_m^{CFT} in terms of the physical parameters of the two-dimensional black hole.

The central charge can be determined using its interpretation as a Casimir energy. (See for instance Ref. [18].) The transformation law of the stress-energy tensor under the change of coordinates (6) is

$$T^{(2)}_{ww} = (\partial_w z)^2 T^{(2)}_{zz} - \frac{c}{12}\{w, z\}(\partial_w z)^2\,, \tag{70}$$

where $w = \tau + \sigma$ and $\{w, z\}$ is the Schwarzian derivative. The vacuum energy is shifted by $l_0 \to l_0 - a^2 c/24$, where l_0 is the eigenvalue of L_0^{CFT} which is associated to the vacuum. This shift corresponds to a Casimir energy $E = -a^2 c\lambda/24$.

The coordinate transformation (6) maps the AdS_2^0 ground state (4) into the AdS_2^+ black hole (5) with mass $m_{bh} = a^2\phi_0\lambda/2$. Because of the duality relation between the gravitational theory and the Dirichlet string we can interpret the previous map as the gravitational theory counterpart of the shift of L_0^{CFT} in CFT_2 and equate the Casimir energy E with m_{bh}. Actually, the equation picks up a minus sign,

$$E = -m_{bh}, \tag{71}$$

because the coordinate transformation (6) maps observers. An observer in the AdS_2^+ vacuum sees the AdS_2^0 vacuum filled with thermal radiation with negative flux [11]. Using Eq. (71) one easily finds

$$c = 12\phi_0\,. \tag{72}$$

Inserting Eq. (72) into Eqs. (69), expressing the string length in terms of λ, $\alpha' = 2\pi/\lambda^2$, and eventually using Eq. (68), Eqs. (69) reduce to Eqs. (9).

The statistical interpretation of the two-dimensional AdS_2 black hole entropy by means of the two-dimensional conformal theory confirms the AdS_2/CFT_2 duality while stressing the peculiarity of the two-dimensional case in the AdS/CFT family. In the case under consideration the duality maps theories that live in spacetimes of identical dimensionality. So the relation is not holographic in the usual sense, because it does not imply the huge reduction of the number of degrees of freedom which is typical of the holographic principle.

This result is also understood through a different, albeit related, argument. The holographic principle puts an upper bound to the information that can be encoded in a spacetime region. This upper bound is given by the Bekenstein-Hawking entropy

$$S_{bh} = \frac{A}{4G}\,, \tag{73}$$

where A is the area of the boundary surrounding the region (the area of the black hole horizon) and G is the Newton constant. In our two-dimensional case the entropy can be written

$$S_{bh} = 2\pi\phi_h = 2\pi r_h \lambda \phi_0\,, \tag{74}$$

where ϕ_h and r_h are the dilaton and the radius evaluated at the horizon, respectively. Since ϕ_0^{-1} plays the role of the two-dimensional Newton constant, the previous relation can be rewritten as $S_{bh} = 2\pi A\lambda/G_2$, where $A = r_h$. This relation is interpreted as an information bound rather than as a holographic bound. Indeed, A is not the area of the boundary – in our case the boundary is a point – but the area of the two-dimensional bulk region $0 < r \leq r_h$. Note that the previous arguments are only valid for the Dirichlet open string. When Neumann boundary conditions are imposed the realization of the AdS/CFT duality is more problematic. In this case a consistent AdS/CFT duality can be realized exclusively by a one-dimensional CFT on the boundary which supports the conventional notion of holography. On the other hand the two-dimensional Stefan-Boltzmann law (9) seems to rule out a realization of the conformal symmetry on the boundary by means of a quantum mechanical system.

The Hawking evaporation process of the two-dimensional AdS_2 black hole [11] has a natural interpretation in the context of the AdS_2/CFT_2 correspondence as well. In Section V we pointed out that in the (t, x) coordinate frame L_0^{AdS} generates dilatations, whereas in the coordinate frame (τ, σ) generates time translations. The coordinate transformation (6) maps the AdS_2^0 ground state (4) into the AdS_2^+ black hole (5). Since positive frequency modes of a quantum field with respect to Killing vector ∂_t are not positive frequency modes with respect to Killing vector ∂_τ, the AdS_2^+ vacuum state appears filled with thermal radiation to an observer in the AdS_2^0 vacuum. The particle spectrum can be obtained calculating the Bogoliubov coefficients between the two vacua [11]. One finds that an observer in the AdS_2^0 vacuum detects a thermal flux of particles with Planck spectrum and temperature (9). The value of the total Hawking flux has been calculated in Ref. [11]. Therefore, the Hawking evaporation effect emerges in the CFT context by requiring that L_0^{AdS} is the generator of time translations.

Up to now we have restricted our considerations to the JT model. Our results can be extended to the general dilaton gravity model (1) provided that its solutions behave asymptotically as in Eq. (25) and Eq. (26). A sufficient condition is that the potential $V(\phi)$ in Eq. (1) behaves for $\phi \to \infty$ as [4]

$$V(\phi) = 2\phi + O\left(\phi^{-2}\right) . \tag{75}$$

One can easily check that in this case the leading term in the weak-coupling expansion (17) describes a free bosonic string. Moreover, Eqs. (9) describe the thermodynamic behavior of the corresponding black solutions at the leading order in the large m_{bh} expansion [4]. Hence, the results obtained in Section VII for the JT model hold for the general model (75) at the leading order in the large m_{bh} expansion.

Let us conclude this section with a remark concerning the relevance of our results for four-dimensional black holes. The two-dimensional dilaton gravity model (1) represents not only a toy model for studying gravitational physics in a simplified context, but describes asymptotically flat four-dimensional black holes in the near-horizon, near-extremal approximation [25] as well. It can be showed that a class of black hole solutions of the effective string theory whose near-horizon behavior is $\mathrm{AdS}_2 \times S^2$, and ϕ varies linearly, exist. Our derivation of the statistical entropy applies straightforwardly to these solutions.

VIII. DISCUSSION

In the previous sections we have been able to work out in detail the correspondence between two-dimensional dilaton gravity on AdS_2 and open strings. Actually, the exact form of the correspondence is perturbative and has only been studied at the leading order in the weak-coupling expansion $\psi \to 0$. We have seen that in this regime two-dimensional dilaton gravity has two degeneration limits that are described by open strings with Dirichlet and Neumann boundary conditions, respectively. Since the description of this degeneracy is based on boundary conditions, it is, however, not completely satisfactory. One would like to understand it in terms of different regions in the parameter space of the theory.

The previous formulation of the AdS/CFT duality is very useful not only because it makes direct contact with the original Maldacena conjecture [1], but also because it can shed some light on several puzzling issues of the AdS/CFT correspondence. The main point of this formulation is the observation that the weak-coupling limit $\psi \to 0$ can be obtained in two different ways. Since $\phi = \phi_0 \lambda r$, the weak-coupling limit can be reached by letting $\phi_0 \to \infty$ at $r = \lambda^{-1}\phi\phi_0^{-1} =$ constant. So we have two weak-coupling regimes: *i)* $r >> 1/\lambda = \sqrt{\alpha'/2\pi}$ and *ii)* $r \sim 1/\lambda = \sqrt{\alpha'/2\pi}$. Note that these weak-coupling limits require that the weak-coupling expansions of the previous sections are written in terms the variables x/ϕ_0 and $(u+v)/\phi_0$, respectively. Since ϕ_0 is equal to 1/12 of the central charge of the CFT, it counts the degrees of freedom and is the two-dimensional analogue of N in the Maldacena conjecture. The limit *i)* corresponds to a one-dimensional CFT on the boundary and describes the excitations of the endpoints of a Neumann open string. The limit *ii)* describes a two-dimensional CFT in the bulk and describes the excitations of a Dirichlet open string.

The duality discussed in our paper has been obtained at the zeroth order in the perturbative expansion (18). Let us discuss qualitatively how the picture is affected by the presence of higher order terms in Eq. (18). Potential terms in the perturbative expansion of the sigma model do not destroy (classical) conformal invariance. Now the model describes open strings propagating in a curved target spacetime. The AdS_2 boundary can be regarded as the (asymptotic) vacuum state of the theory and the string field is expanded in normal modes around this vacuum. Imposing Dirichlet boundary conditions we find the correspondence between the gravitational modes and the string modes, Eq. (64). The theory can then be expressed at any order as a function of the first order gravitational modes $M_{1,m}$, $\psi_{1,m}$ or, alternatively, as a function of the string modes α^μ_m. The potential term at a given order gives the interaction term for the modes. Obviously, dealing with an interacting theory, the relation between the stress-energy tensor and the Virasoro generators (39) is not valid. Though the first order gravitational modes define uniquely the stress-energy tensor, the relation of the latter with the string modes is more complicated than Eq. (39). Consequently, we expect the central charge to be different from the central charge of the free theory, $c = 12\phi_0$, and the thermodynamic relations (9) to be altered. This is no surprise. Higher order (off-shell) corrections to the free theory on the boundary induce (quantum) corrections to the black hole geometry that affect the derivation of the thermodynamic relations (9). Calculating at a given perturbative order the black hole geometry and the central charge one could find out how the statistical derivation of the entropy is affected by the presence of the interaction terms.

The existence of two degeneration limits of the weak-coupled dilaton gravity theory clarifies some controversial issues of the two-dimensional AdS/CFT correspondence. The two-dimensional CFT with Dirichlet boundary conditions gives a consistent explanation of the features of the dilaton gravity theory. We might conclude that the microscopic dynamics of two-dimensional black holes is fully captured by a two-dimensional CFT. This conclusion is not completely satisfactory, however, because a one-dimensional CFT living on the boundary of AdS_2 emerges in our picture as well. The role of the AdS_2/CFT_1 correspondence, and its relation to the AdS_2/CFT_2 duality, are not yet fully understood and deserve further investigations. In this respect, the results of this paper seem to give contradictory indications. Although a CFT_1 fits naturally in our scheme, it is indeed very difficult to understand how it could explain the energy-temperature relation (9).

It has been proposed [7] that conformal mechanics, possibly in the form of large N Calogero models, describes the ground state of the two-dimensional black holes arising as near-horizon geometry of the four-dimensional Reissner-Nordström black hole, which is characterized by a constant dilaton. If we could extend this proposal to our case, which is characterized by a non-constant dilaton, the conformal mechanics would describe the ground state, whereas the two-dimensional CFT would describe the black hole excitations [9,26,27]. However, the existence of a mass gap that separates the ground state from the continuous part of the spectrum – typical of the Reissner-Nordström-like black holes but absent in the JT case – seems a crucial missing ingredient to make this proposal feasible.

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